

Study of Diversity in Phyllotactic Patterns

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#Agenda

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- Phyllotaxis

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- Anatomy Study

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- Anatomy Study
- Spadix Projection

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- Projection Unwarping

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- Projection Unwarping
- Microscopy

#Introduction

Phyllotaxis

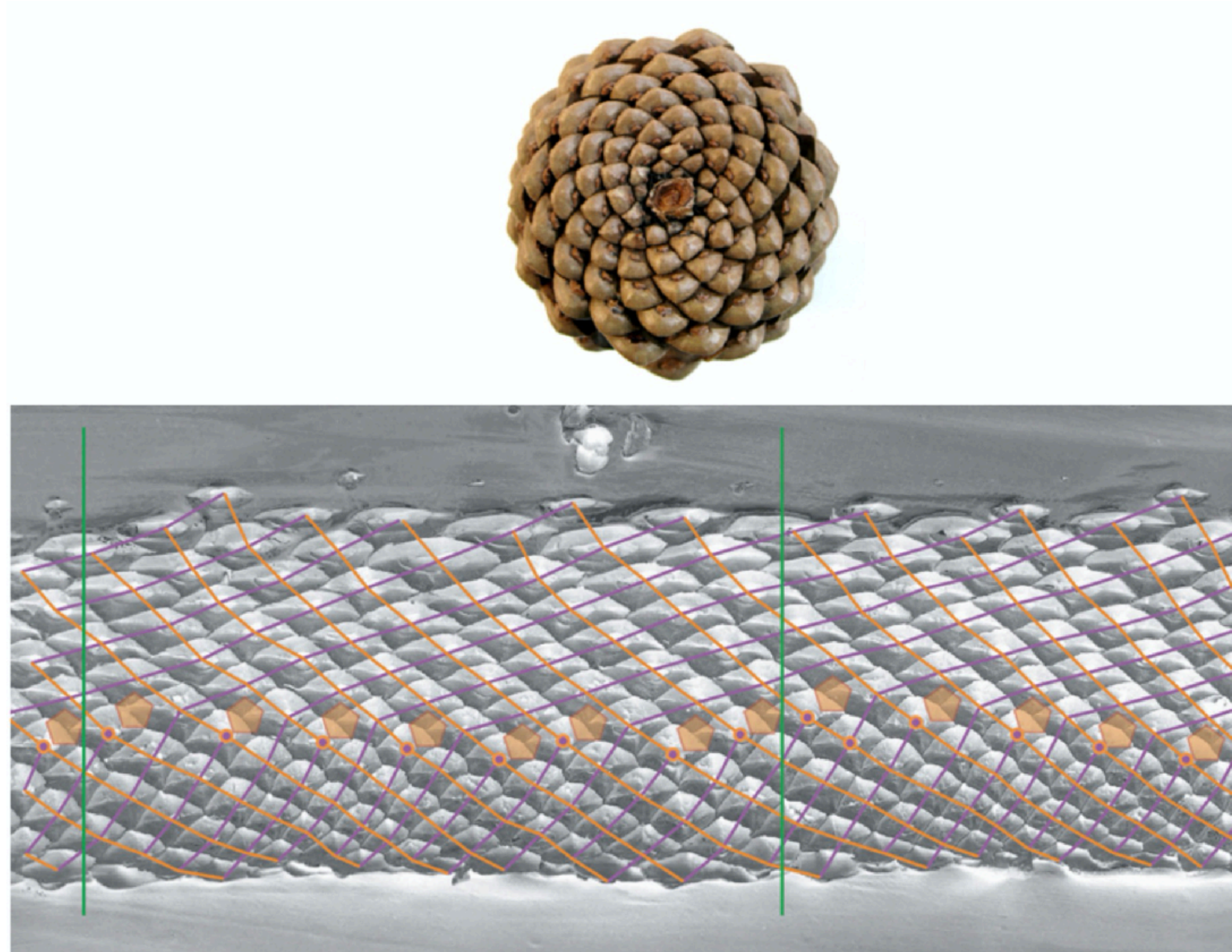
Phyllotaxis

- Phyllotaxis is the arrangement of lateral organs in plants
- This arrangement, commonly in form of spirals or helices, is called Parastichies.
- The number of parastichies is very often observed to be a pair of successive Fibonacci or Lucas numbers.



Phyllotaxis

- Phyllotaxis attempts to explain the origin of the emergent patterns, transitions, and irregularities.

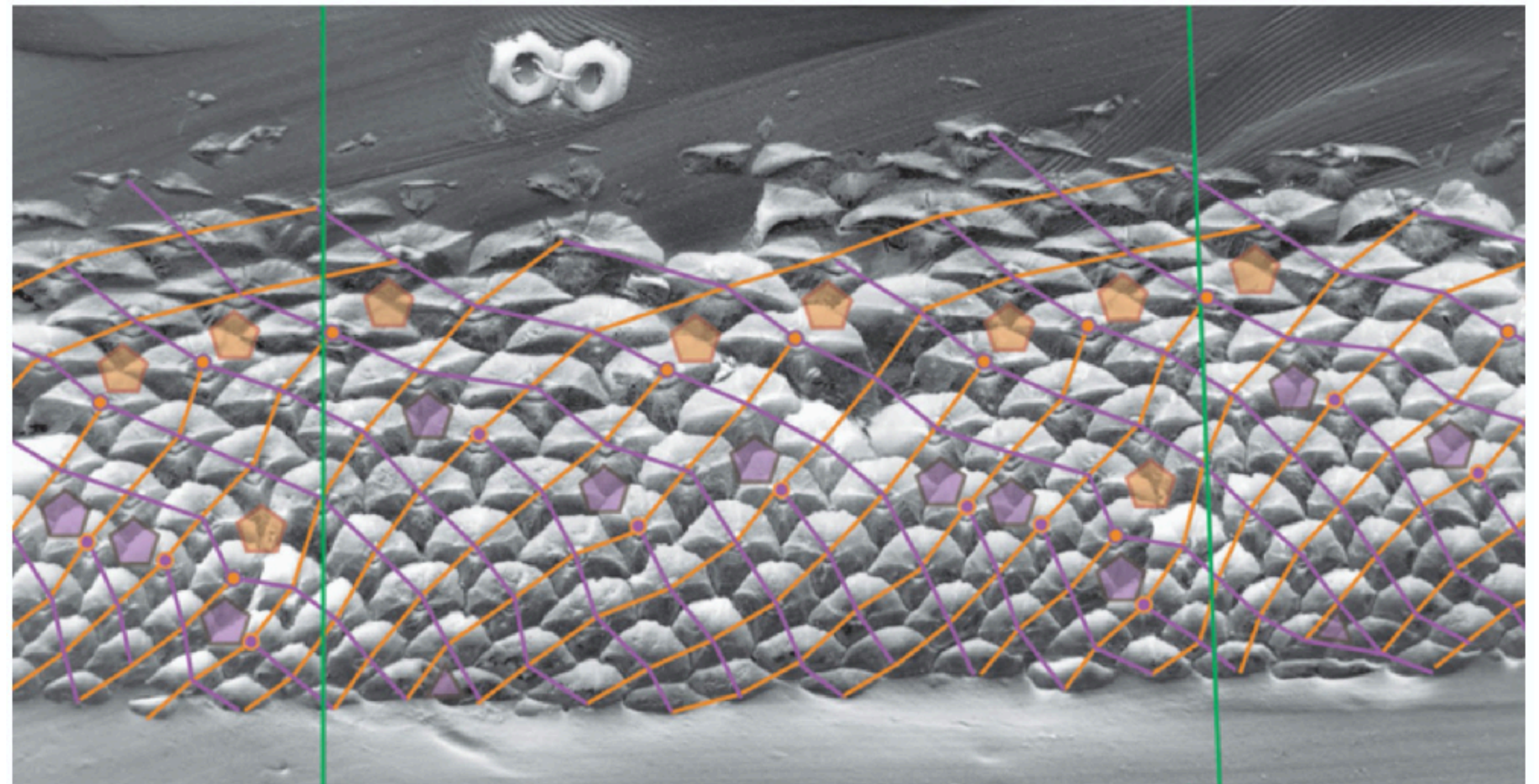


Phyllotaxy of a regular *Pinus nigra* cone.

Golé et. al. Fibonacci or quasi-symmetric phyllotaxis. Part I: Why?
2016, Acta Societatis Botanicorum Poloniae

Phyllotaxis

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Phyllotaxy of an irregular *Pinus nigra* cone.

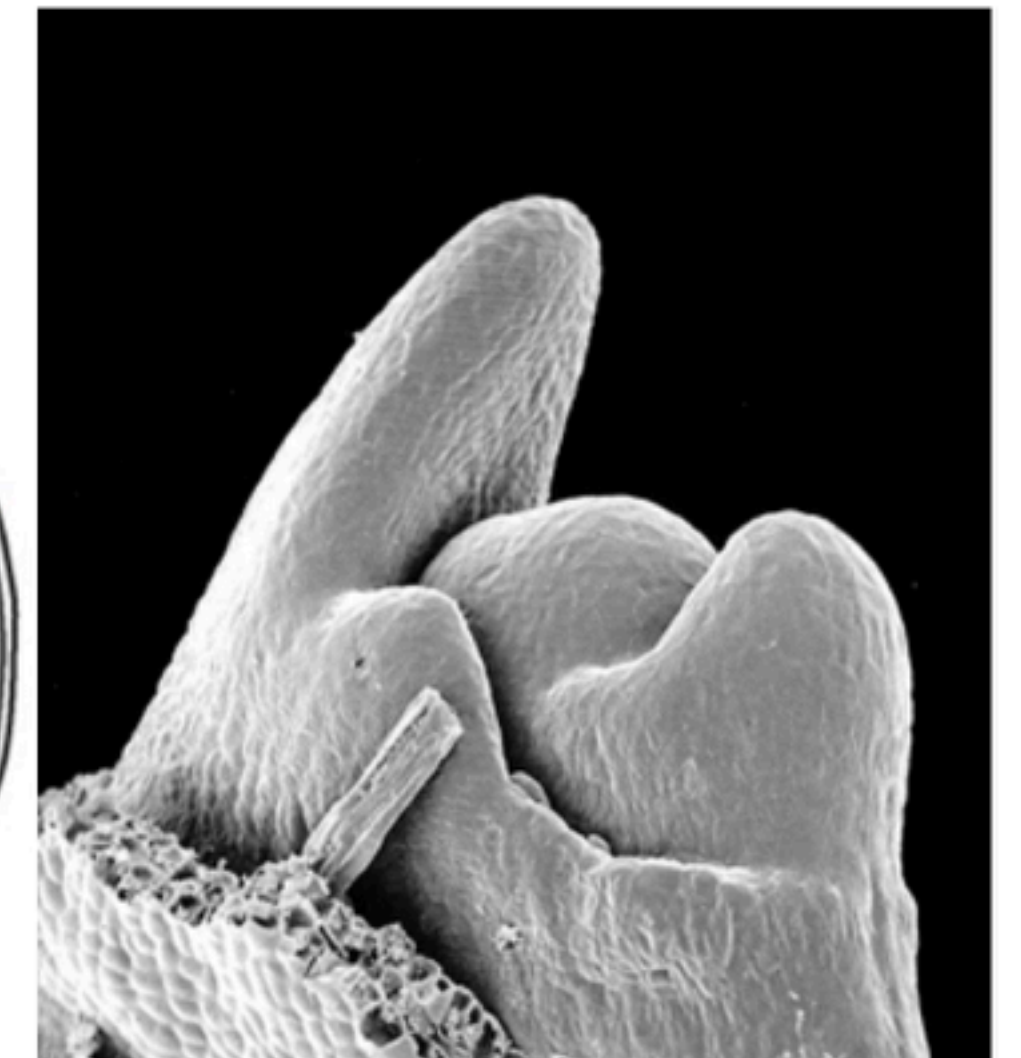
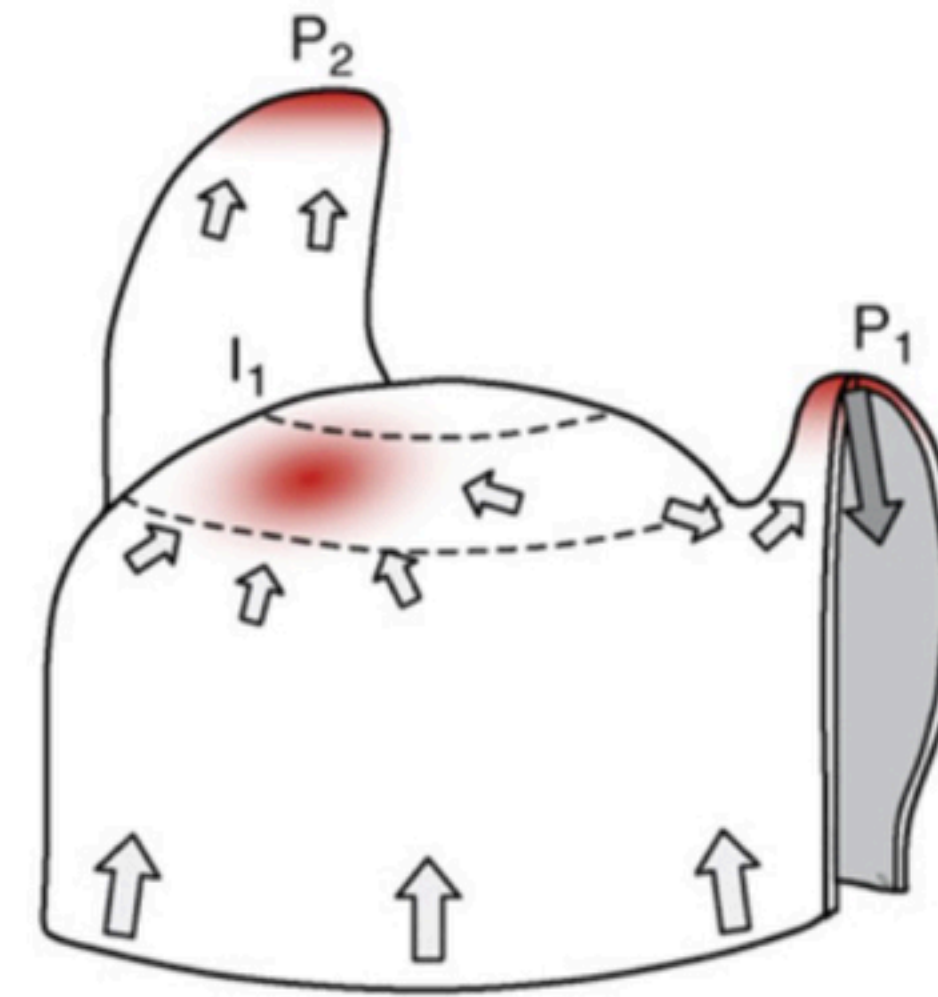
*Golé et. al. Fibonacci or quasi-symmetric phyllotaxis. Part I: Why?
2016, Acta Societatis Botanicorum Poloniae*

#Central Question

**What influences the regularity of
Parastichies?**

Developmental Dynamics of Phyllotactic Patterns

- Phyllotactic patterns are established through the activity in *shoot apical meristem*.
- Plant hormones *auxin* and *auxin efflux transporter*, **PIN**, interact in a process called *polarised auxin transport*.
- **PIN** amplifies disparity in auxin concentration, hence making it non-uniform.

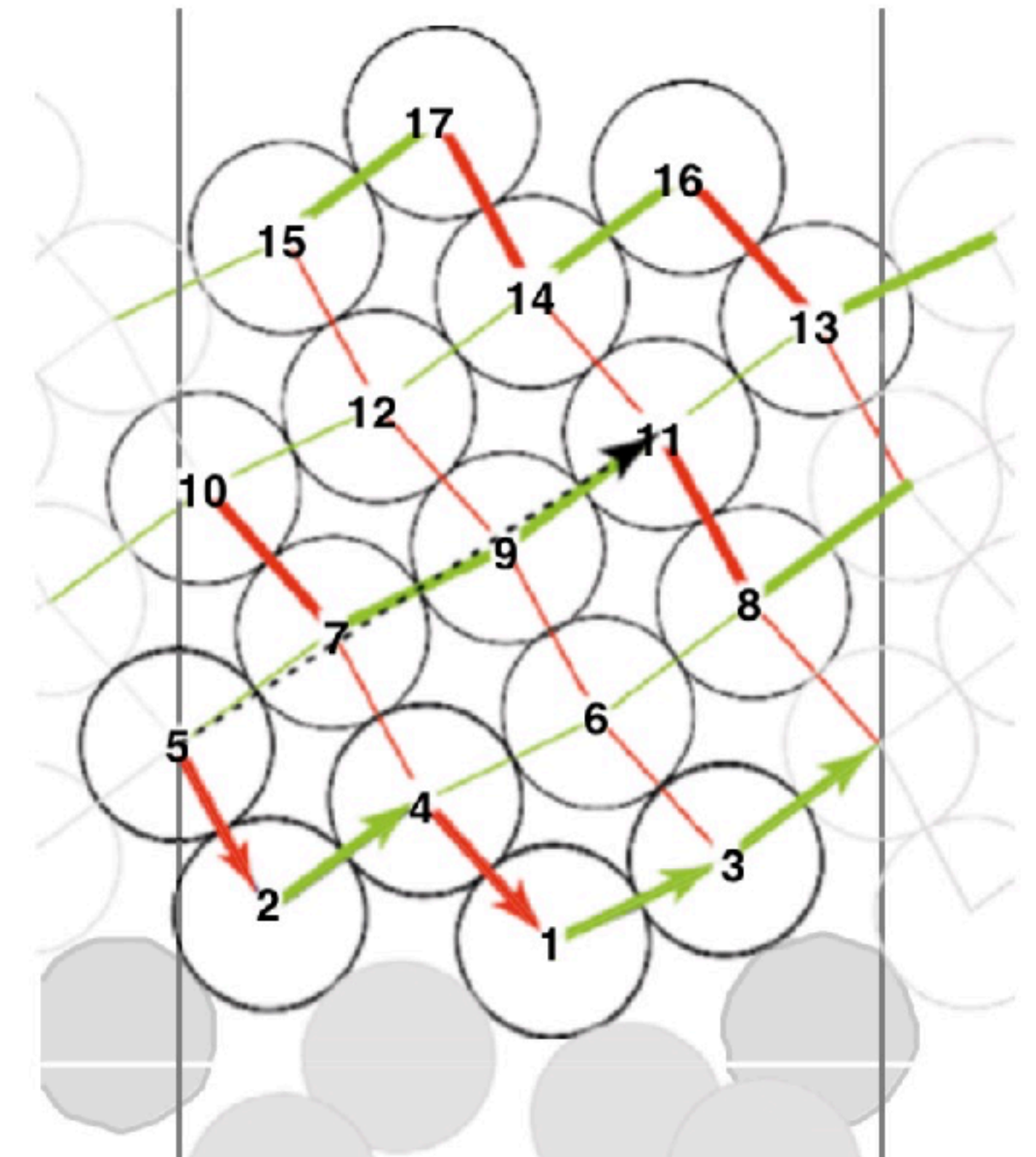


L. Polar auxin transport gradient.

R. Electron microscopy of Muehlenbeckia apex.

Disk Stacking Model

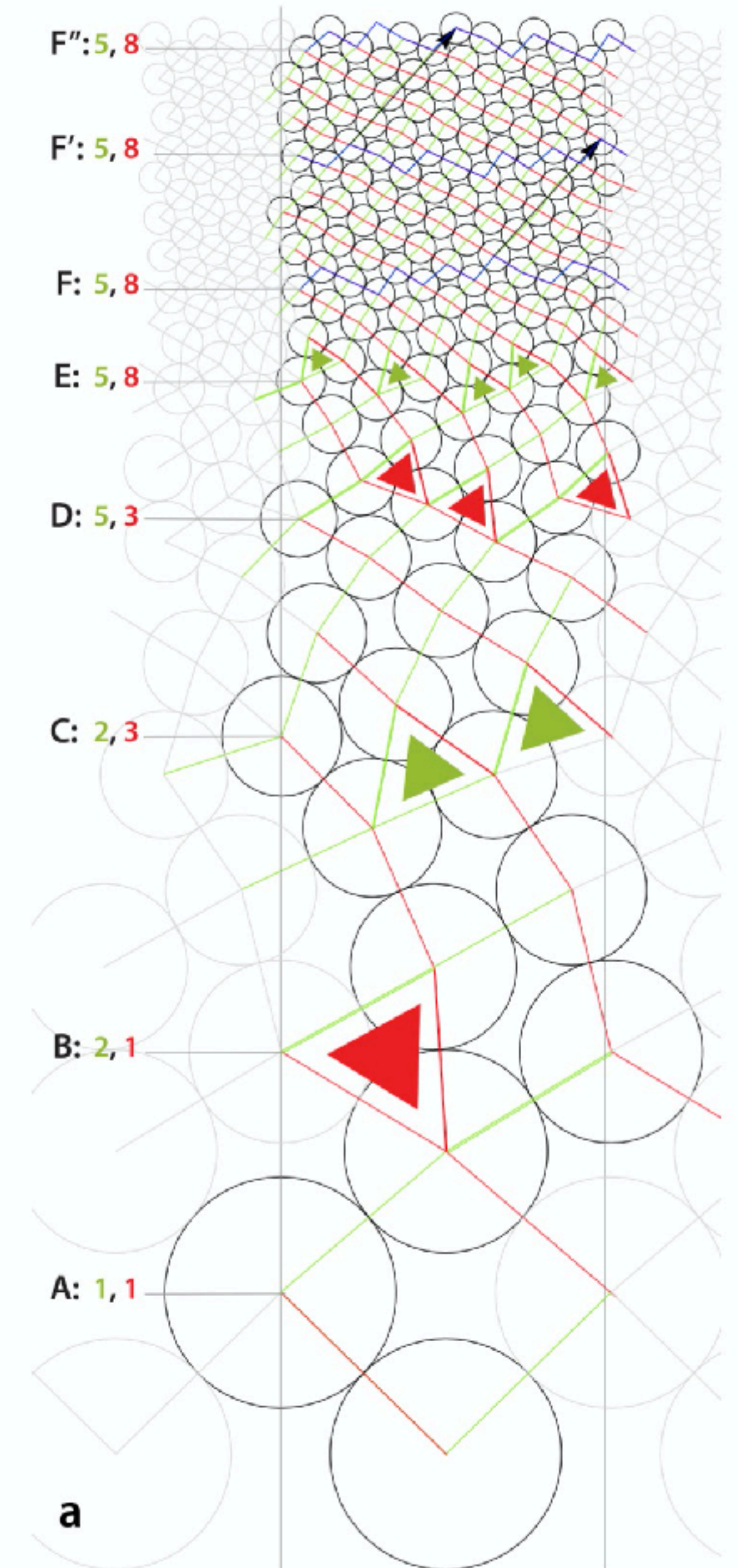
- Provides a geometric mechanisms of phyllotactic transitions.
- The stem is modelled on a cylinder, and the *primordia* are represented as disks.
- New disks are added on the surface in the lowest available space without overlapping existing disks.



#Introduction

Pattern Irregularity, Parastichies, and Disk Stacking Model

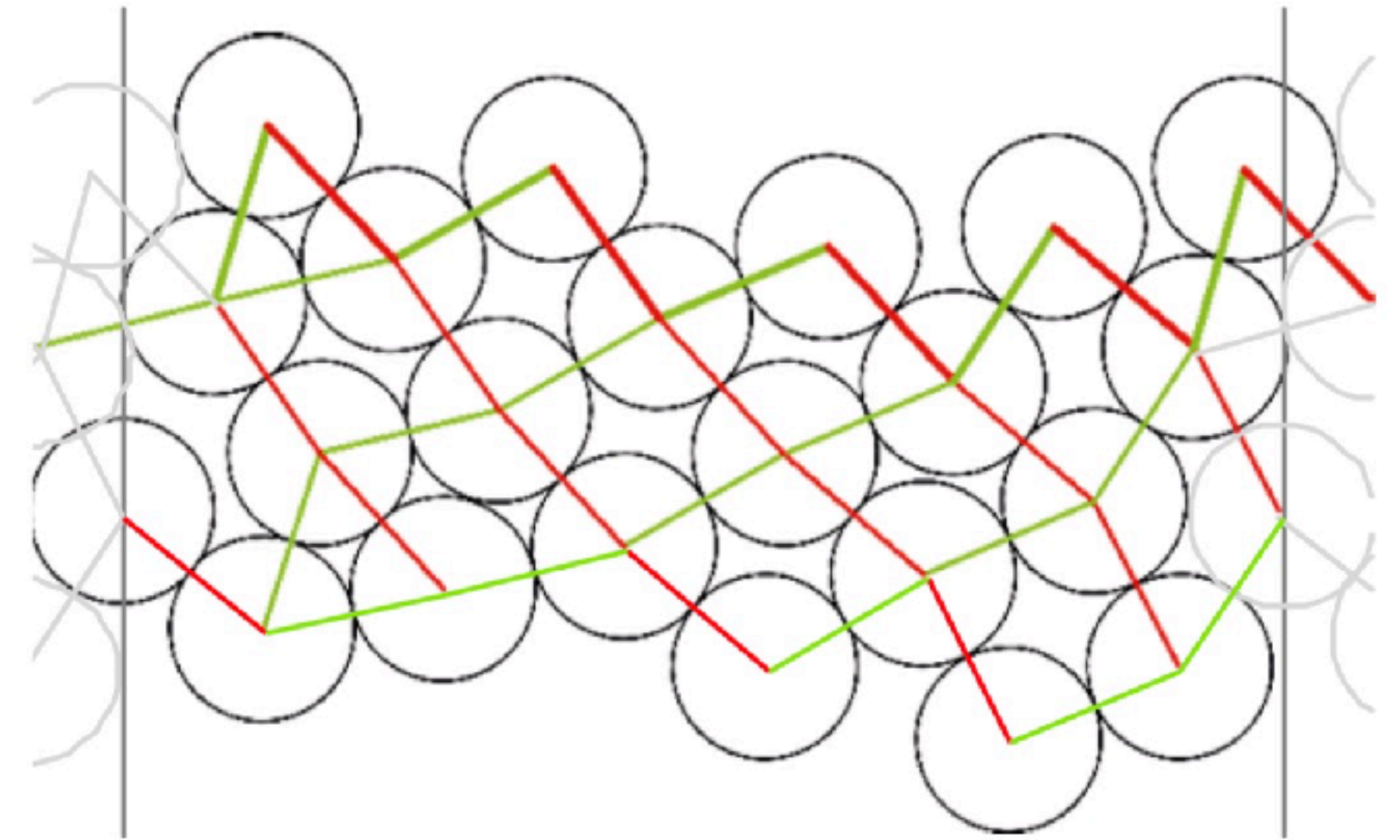
- An iterative version of Disk Stacking Model is obtained by repeated addition of "primordial fronts".
- Starting with a $1, 1$ front, the pattern converges to *Fibonacci transitions*.



Douady et. al. Fibonacci or quasi-symmetric phyllotaxis. Part II: botanical observations
2016, Acta Societatis Botanicorum Poloniae

Deviation of Parastichies Transition from *Fibonacci* Transition

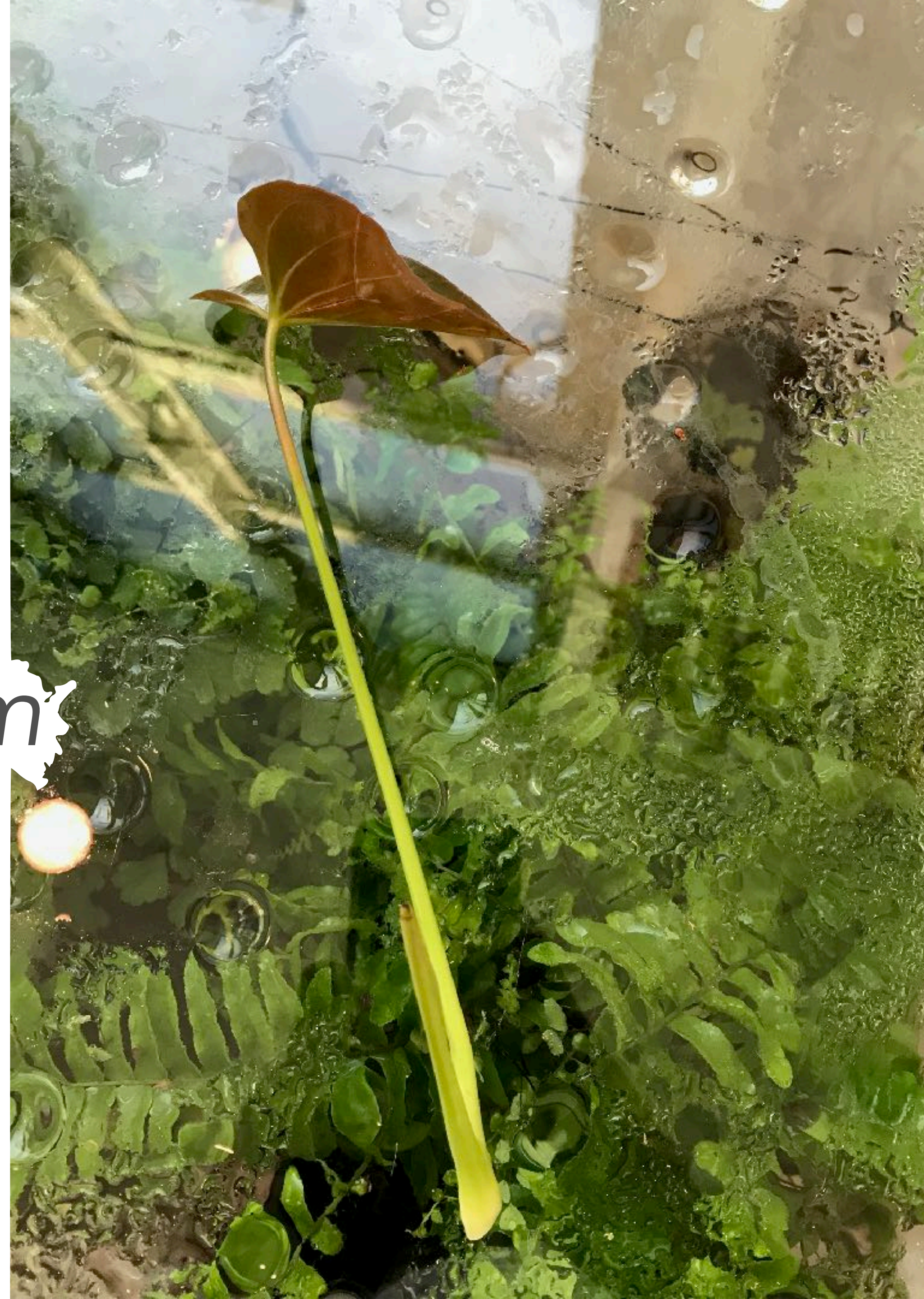
- The model fails to converge at a Fibonacci Transition when either initial fronts are too irregular and/or the disks shrink too quickly.
- A predominance of quasi-symmetric patterns is observed in these cases.



Drift toward symmetry.
An irregular (5, 3)-front yields a (5, 5)-front,

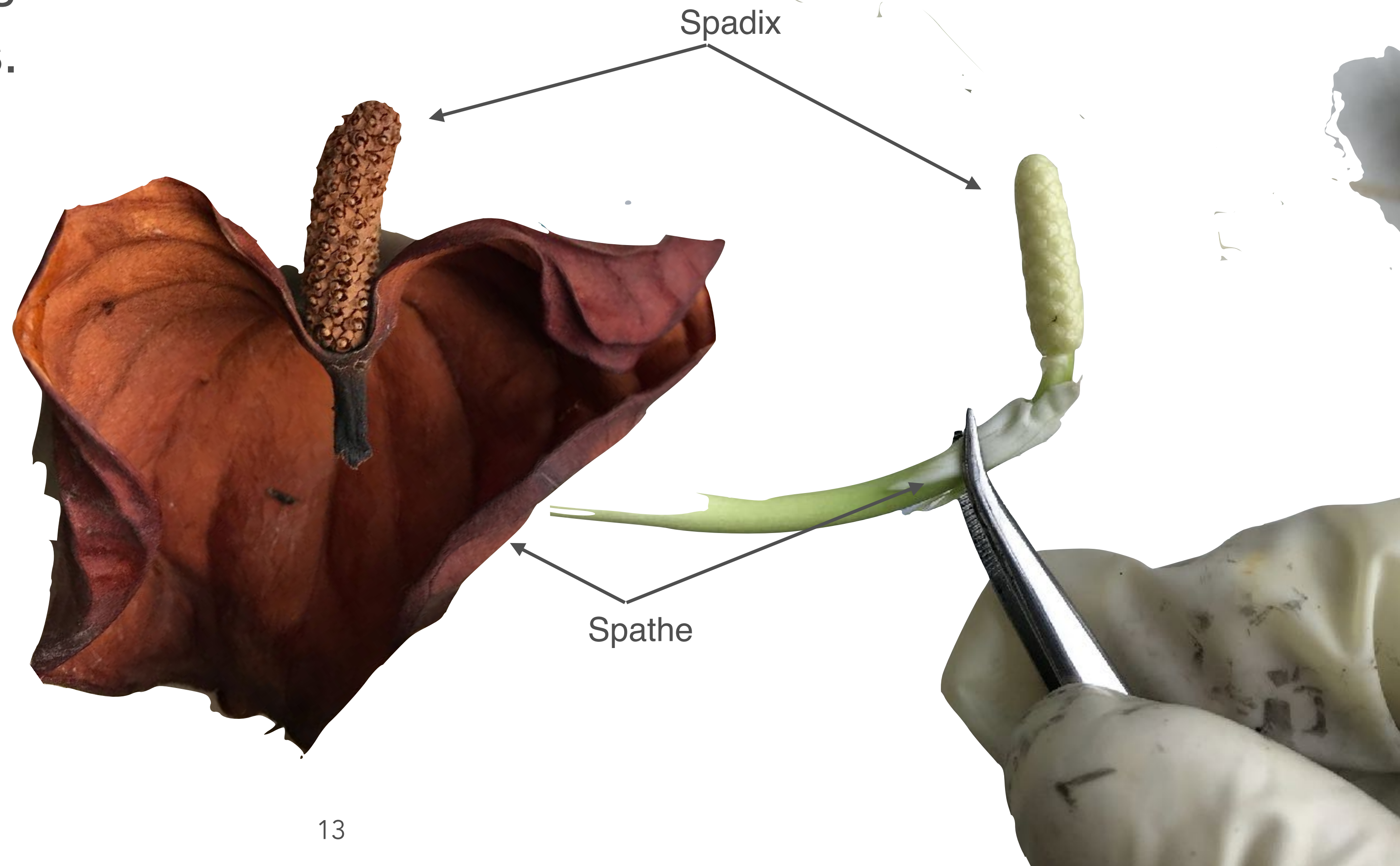
#Anatomy

Phyllotaxy of *Anthurium andraeanum*



Phyllotaxis of *Anthurium andreaeanum*

- Flowering plant species in the Anthurium genus.
- Has protruding elongation called a Spadix.



10x



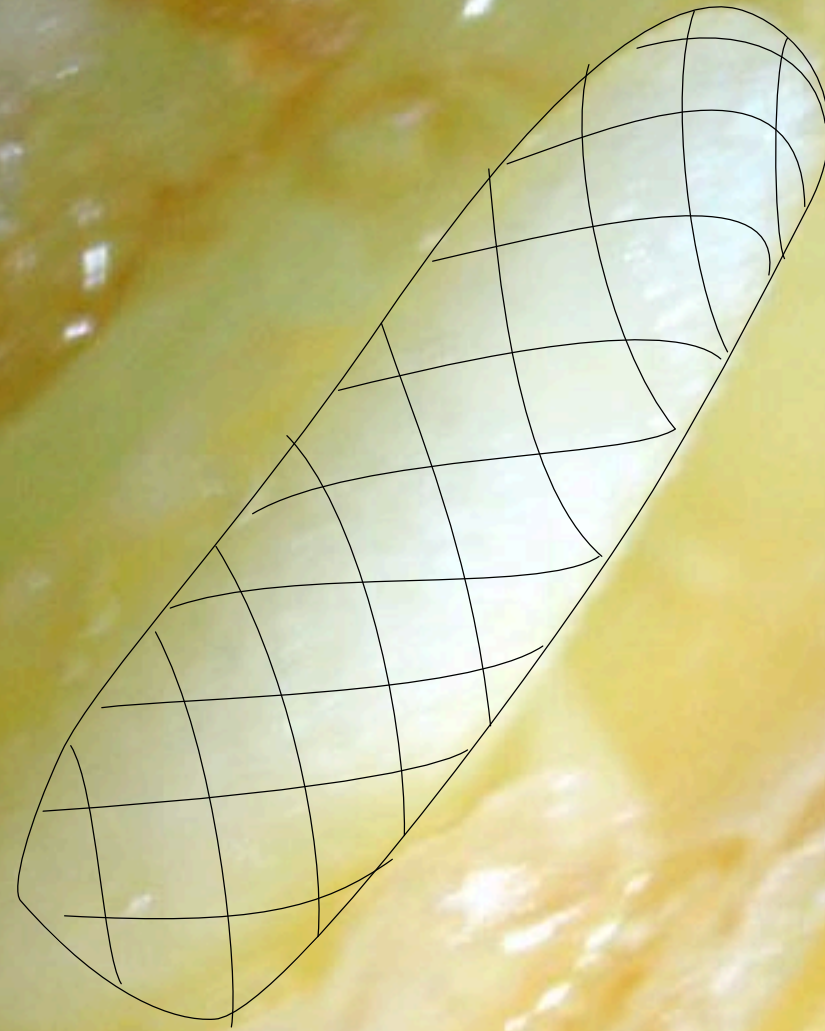
20x



30x



30x

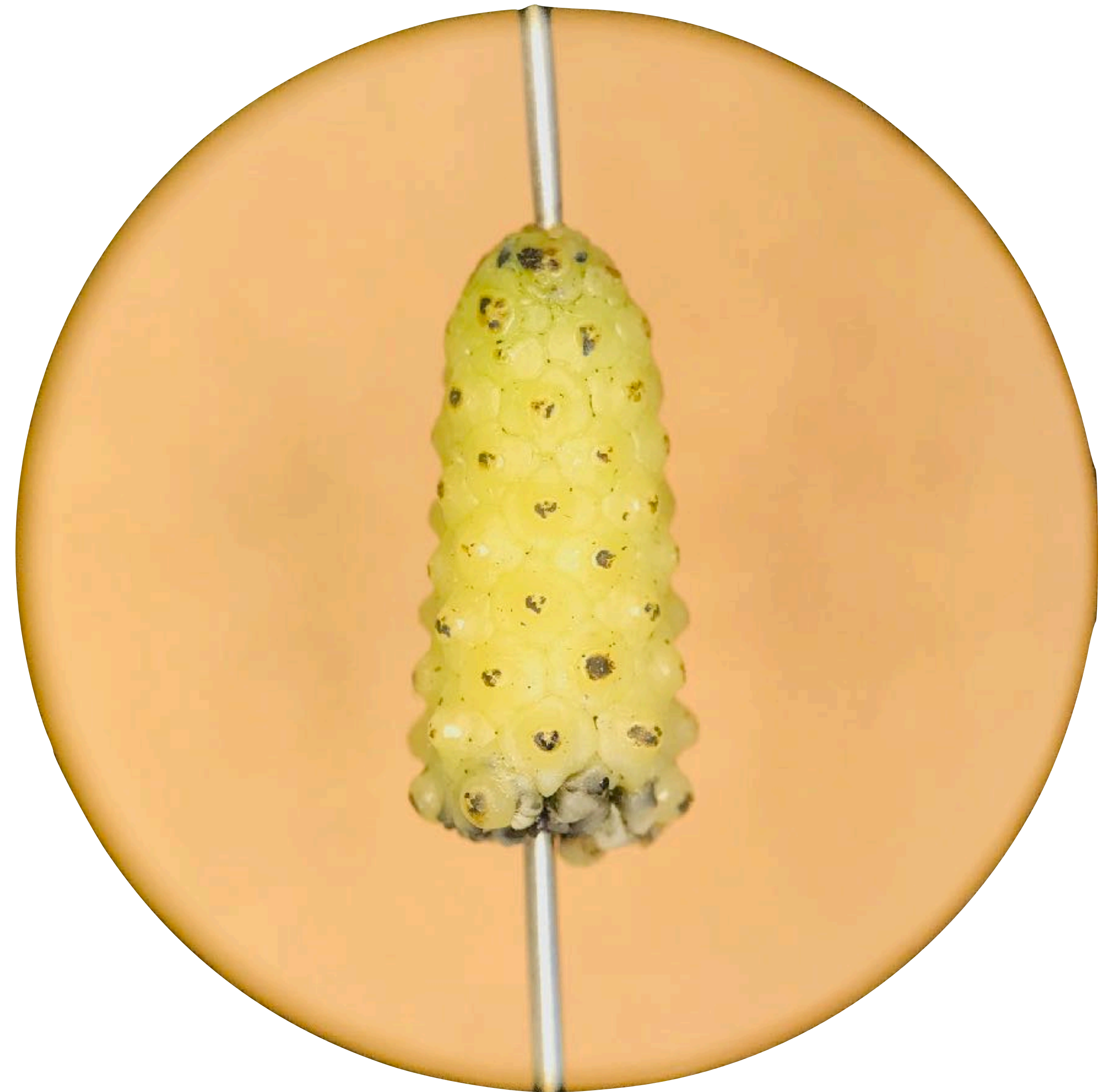


#Phyllotaxy

Unrolling Spadix

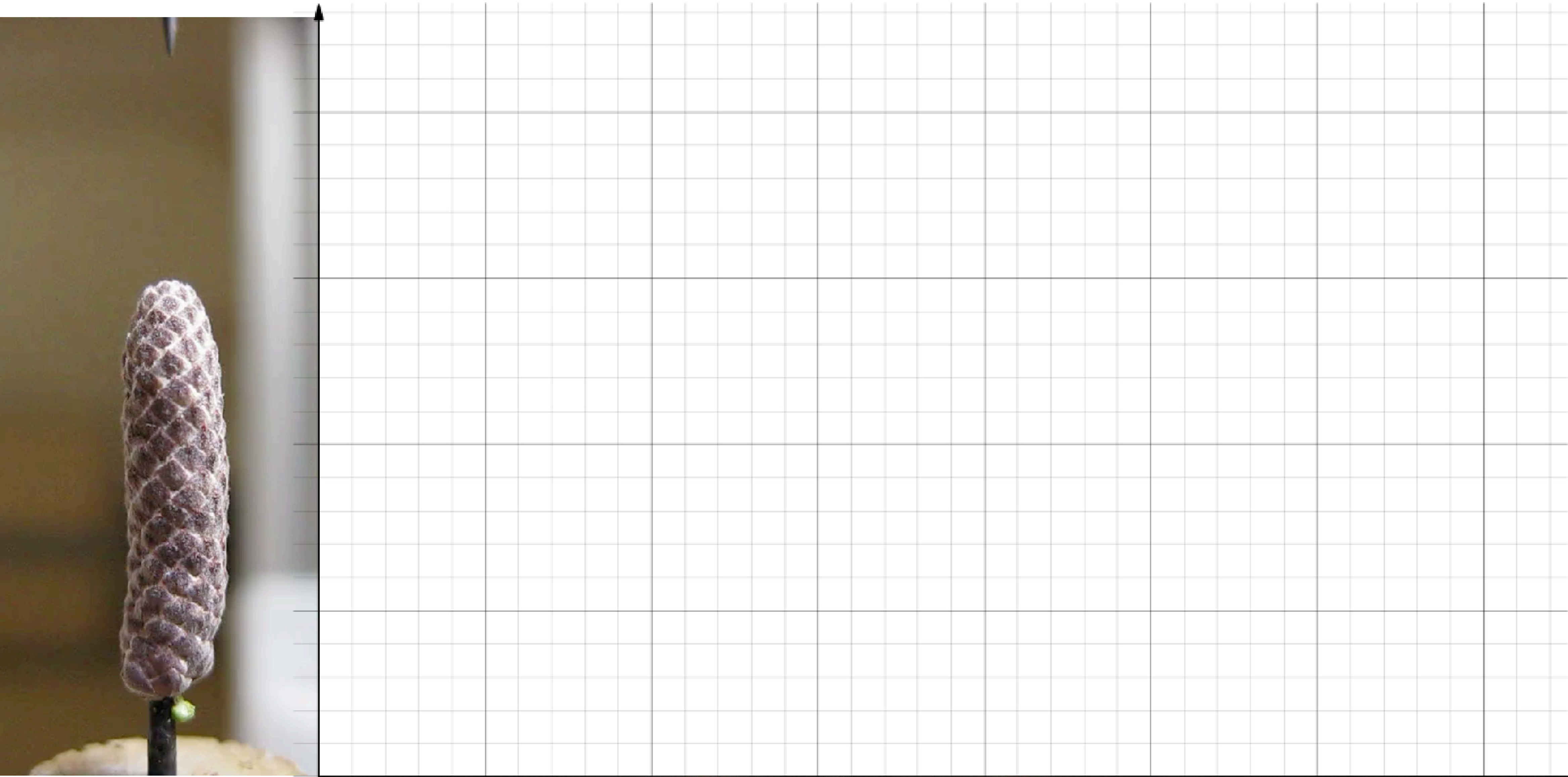
Unrolling Spadix

- A Spadix is attached between rotating needles.
- We capture the video of this rotation and use scan-line unrolling to collapse frames into a single image.



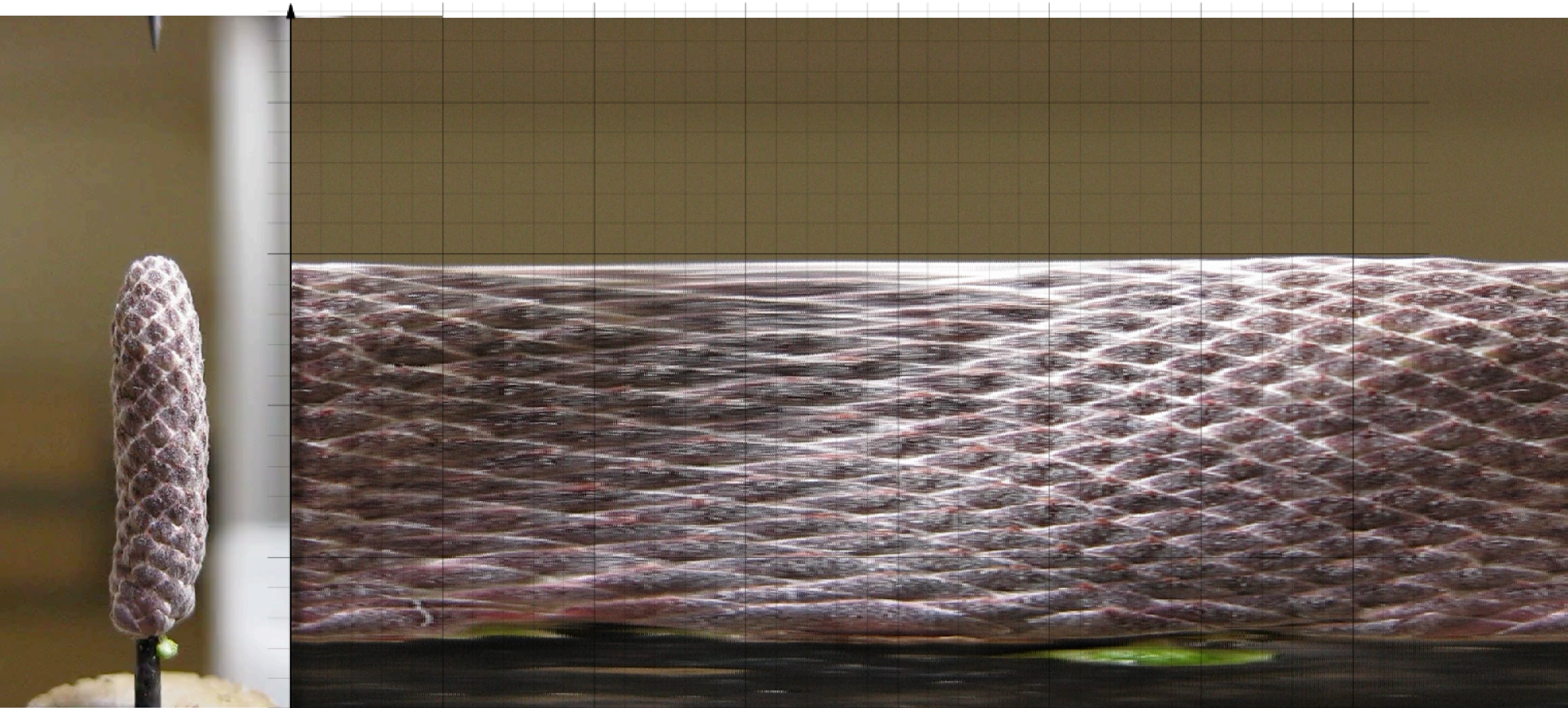
#Phyllotaxy

Scan-Line Unrolling



#Phyllotaxy

Scan-Line Unrolling



#Unwarping

Projection Distortions

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Unrolling a Cone

- Dilation and contraction of surface elements.
- These distortions can be attributed to difference in angular velocity of topology.



#Projection Distortions

Unrolling a Tube

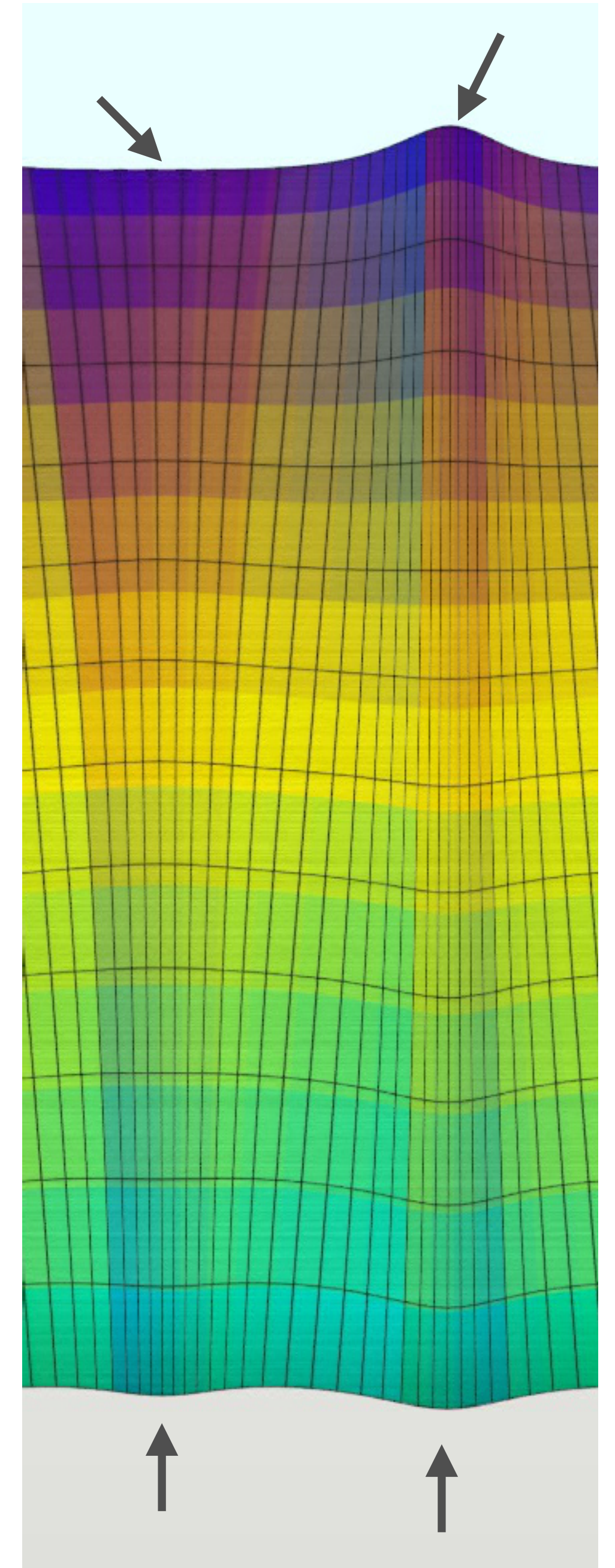
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#Projection Distortions

Simulated Geometries

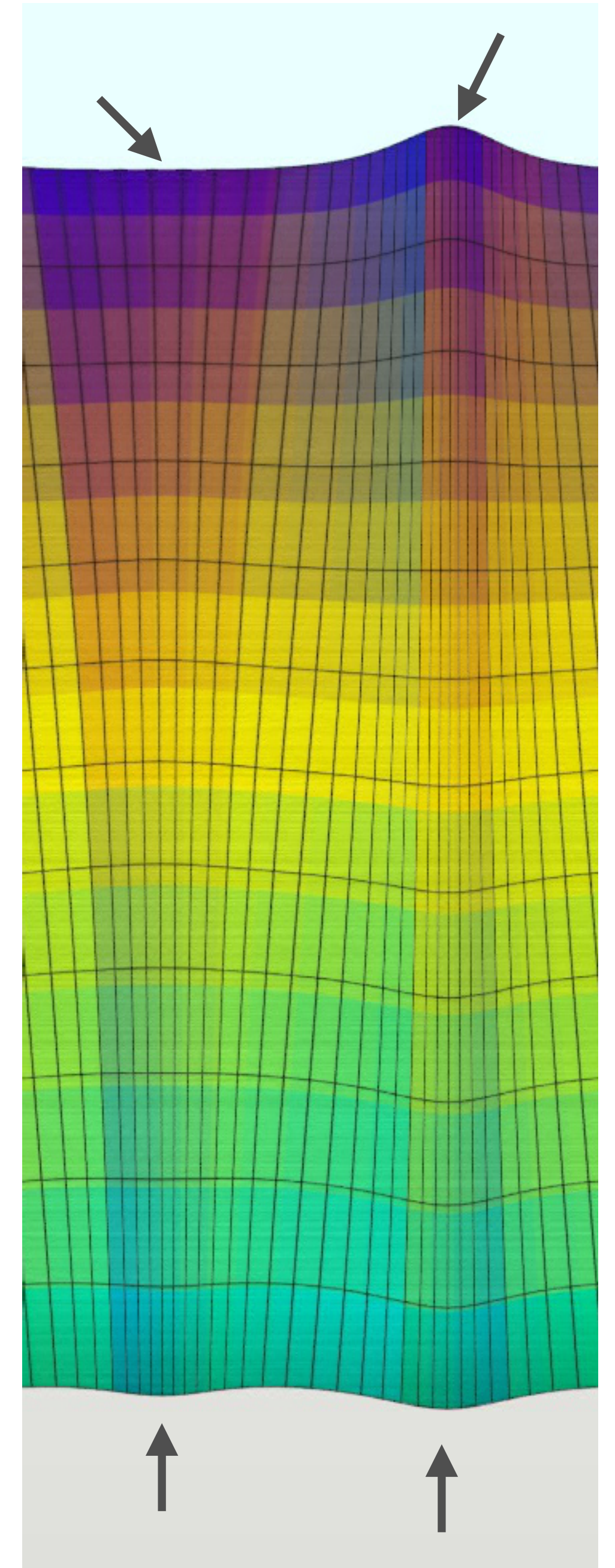
- Using Simulated Geometries, the distortions are easier to spot.
- These geometries also helped to model the problem.



#Projection Distortions

Simulated Geometries

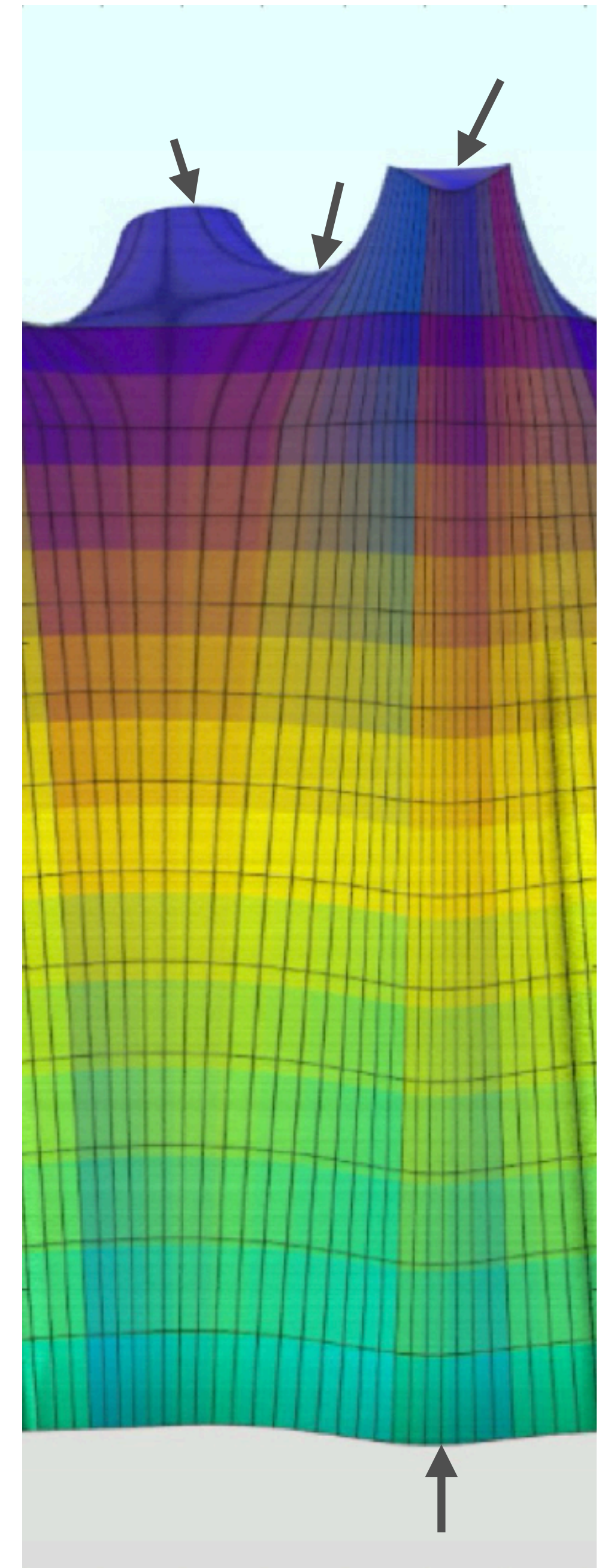
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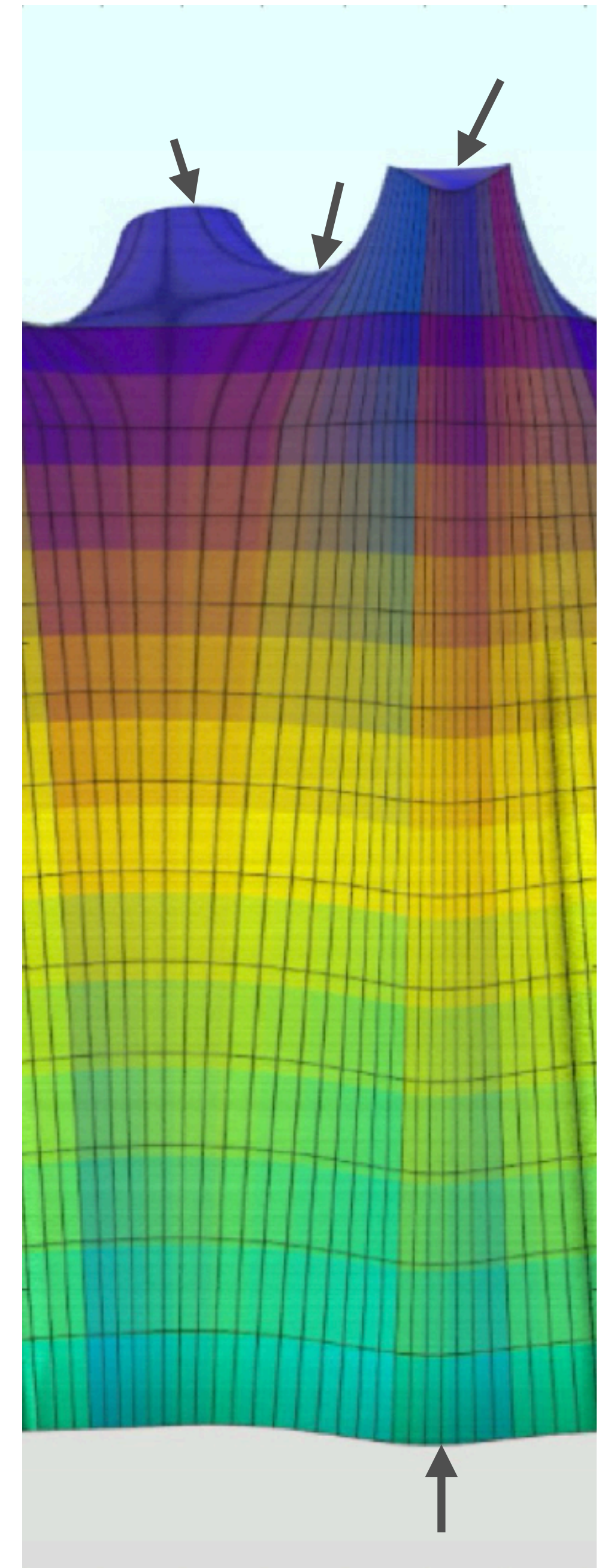
- The distortion is more pronounced for a non-standard cylindrical surface.



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Simulated Geometries

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#Unwarping

Unwarping Distorted Projections

#Unwarping

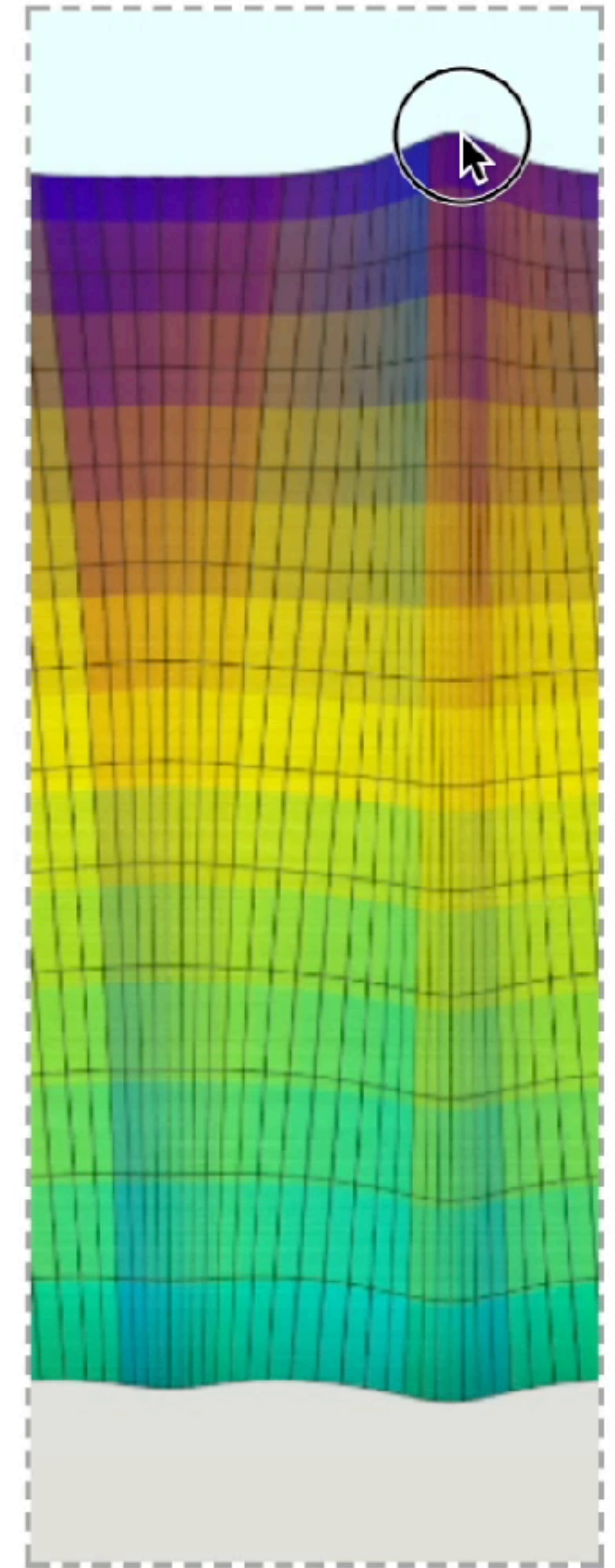
Unwarping Projections

- For better analysis of Parastichies, it's important to unwrap its projection.
- Here, we wish to find an appropriate image transformation to recover original element dimensions.

#Unwarping Attempt 1

Fixing Distortions using Affine Transformation

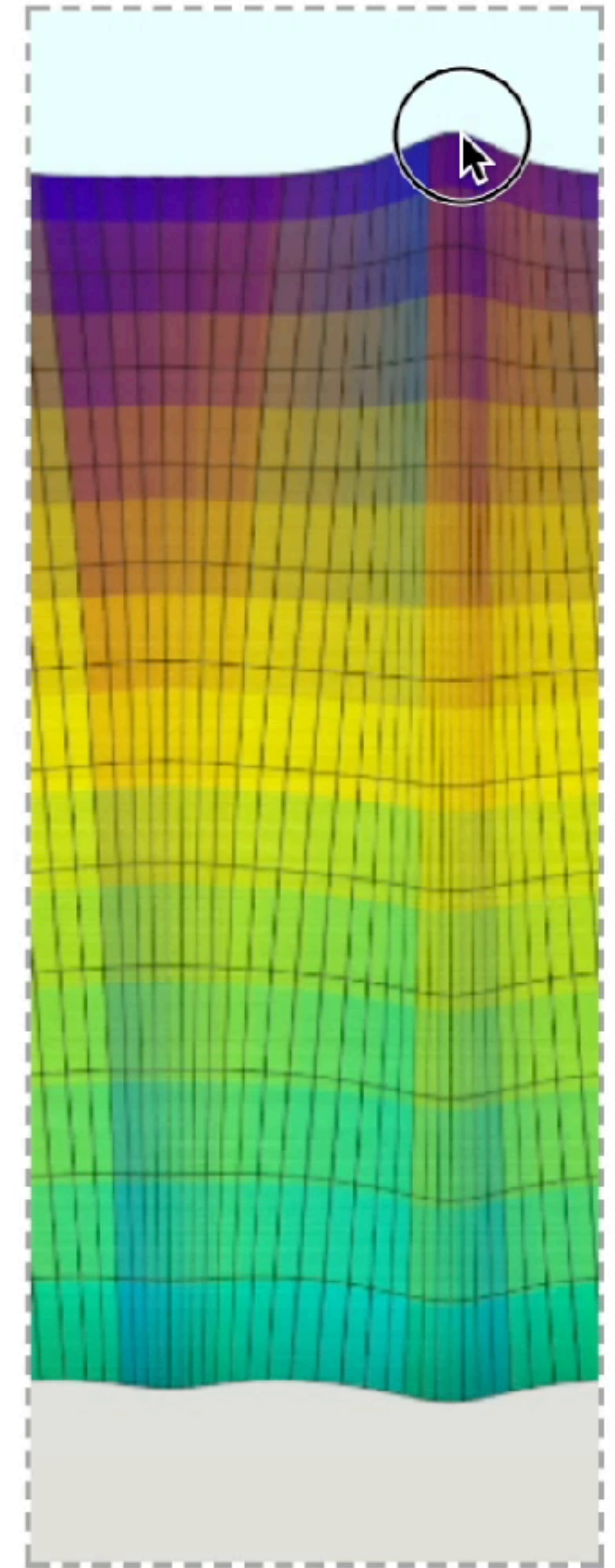
- Apply a grid based piece-wise affine transformation on image to fix the distortions.
- Developed a tool for grid warping input.



#Unwarping Attempt 1

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#Unwarping Attempt 1

Fixing Distortions using Affine Transformation

- This approach isn't very convenient or accurate.
- Find better way!

#Unwarping Attempt 2

Fixing Distortion by Cylinder Unrolling Model

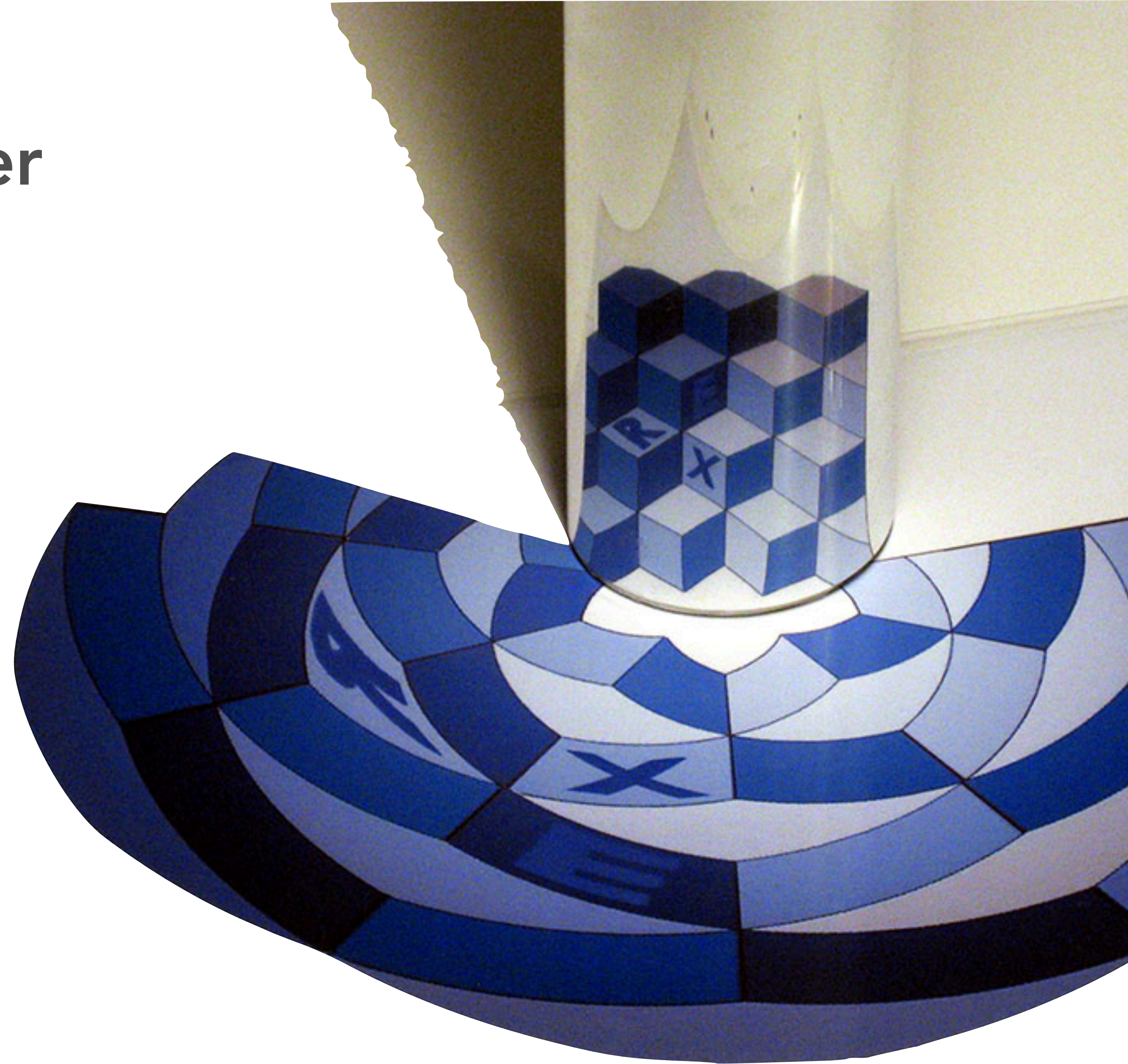
- Approach the unwarped image's distortion as a projection of curve on cylinder to a plane.
- The distortion can be fixed using inverse projection.



#Unwarping Attempt 2

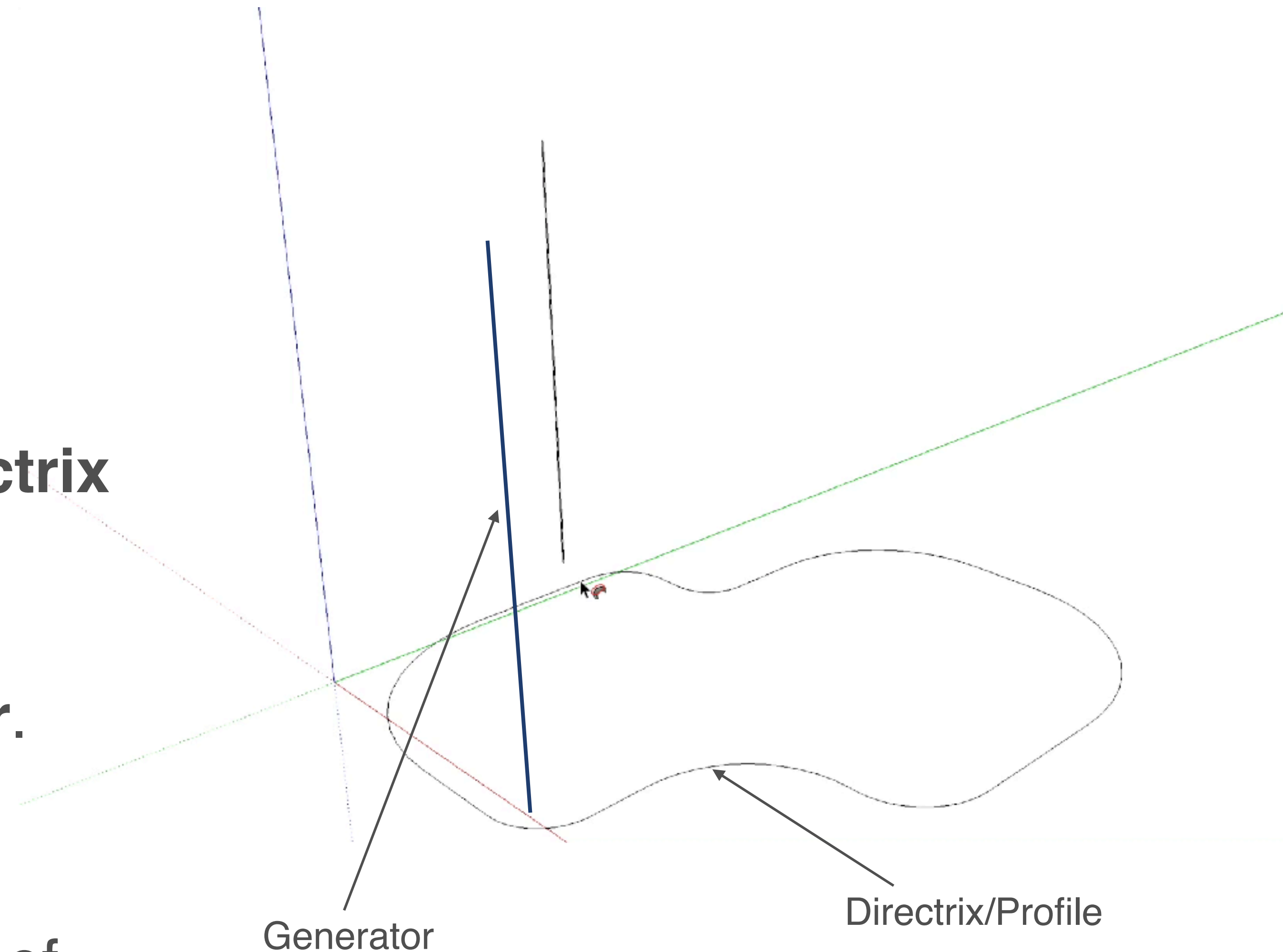
Curve Unrolling from Cylinder

- Consider a Cylinder having a Curve drawn on its surface.
- What happens to the curve when this cylinder is unrolled?
- Conversely, this can be seen as equivalent to curves being projected *onto* a Cylinder.



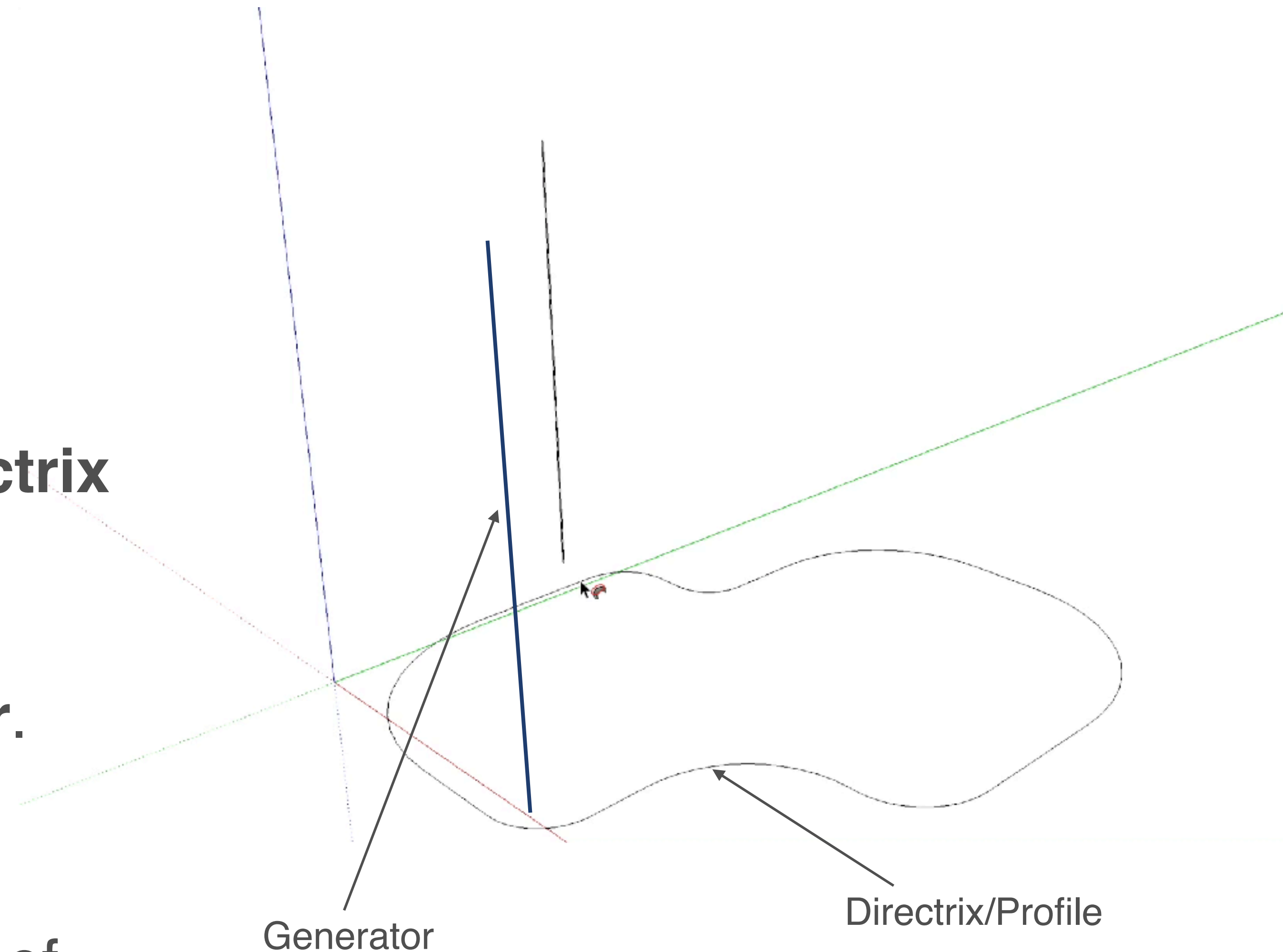
Realisation

- A cylinder is a *surface swept* by a straight line moving along a **planar curve**.
- This planar curve is called the **directrix** of the cylinder.
- The moving line is called **generator**.
- If *directrix curve* is perpendicular to *generator line*, we call it the **profile** of cylinder.



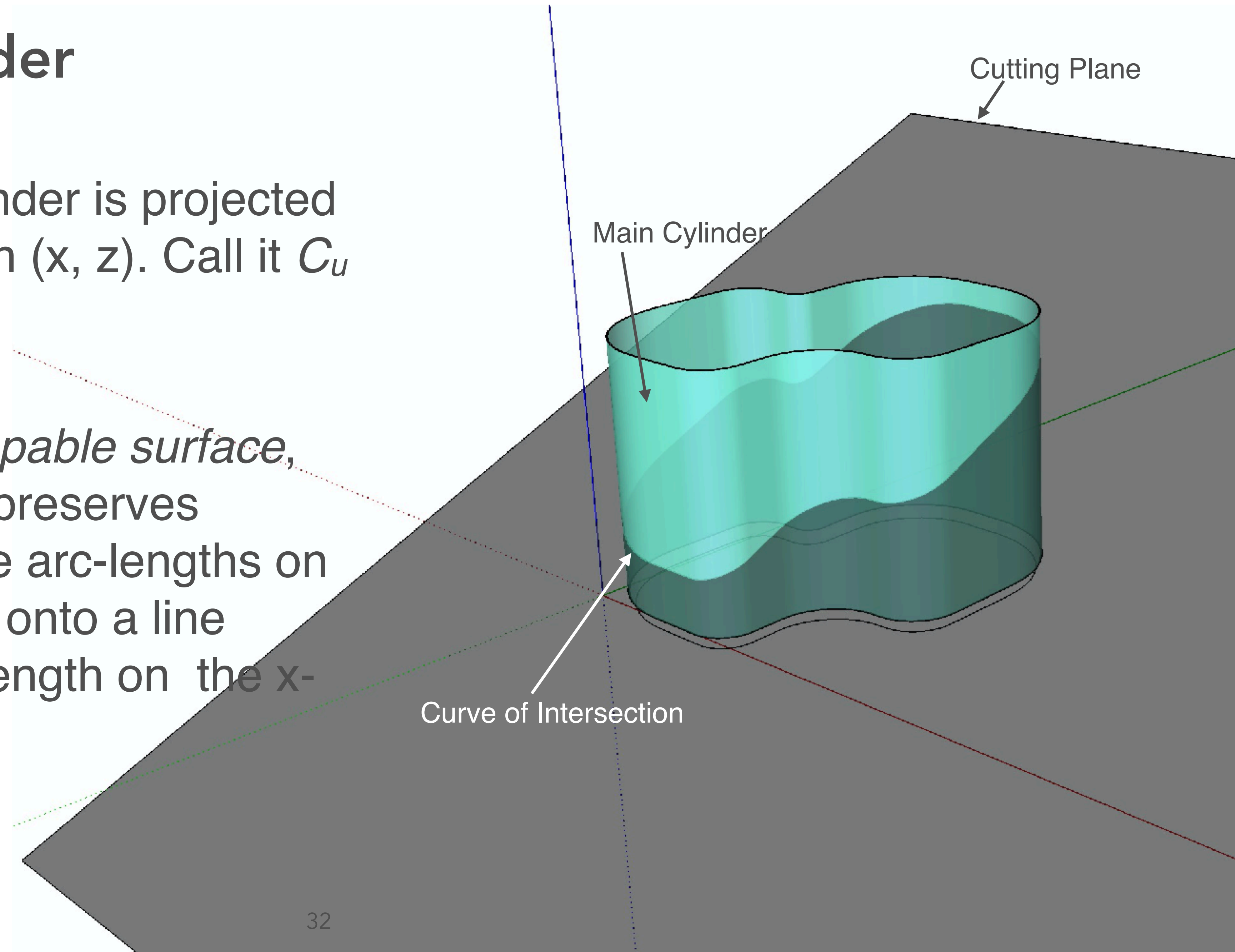
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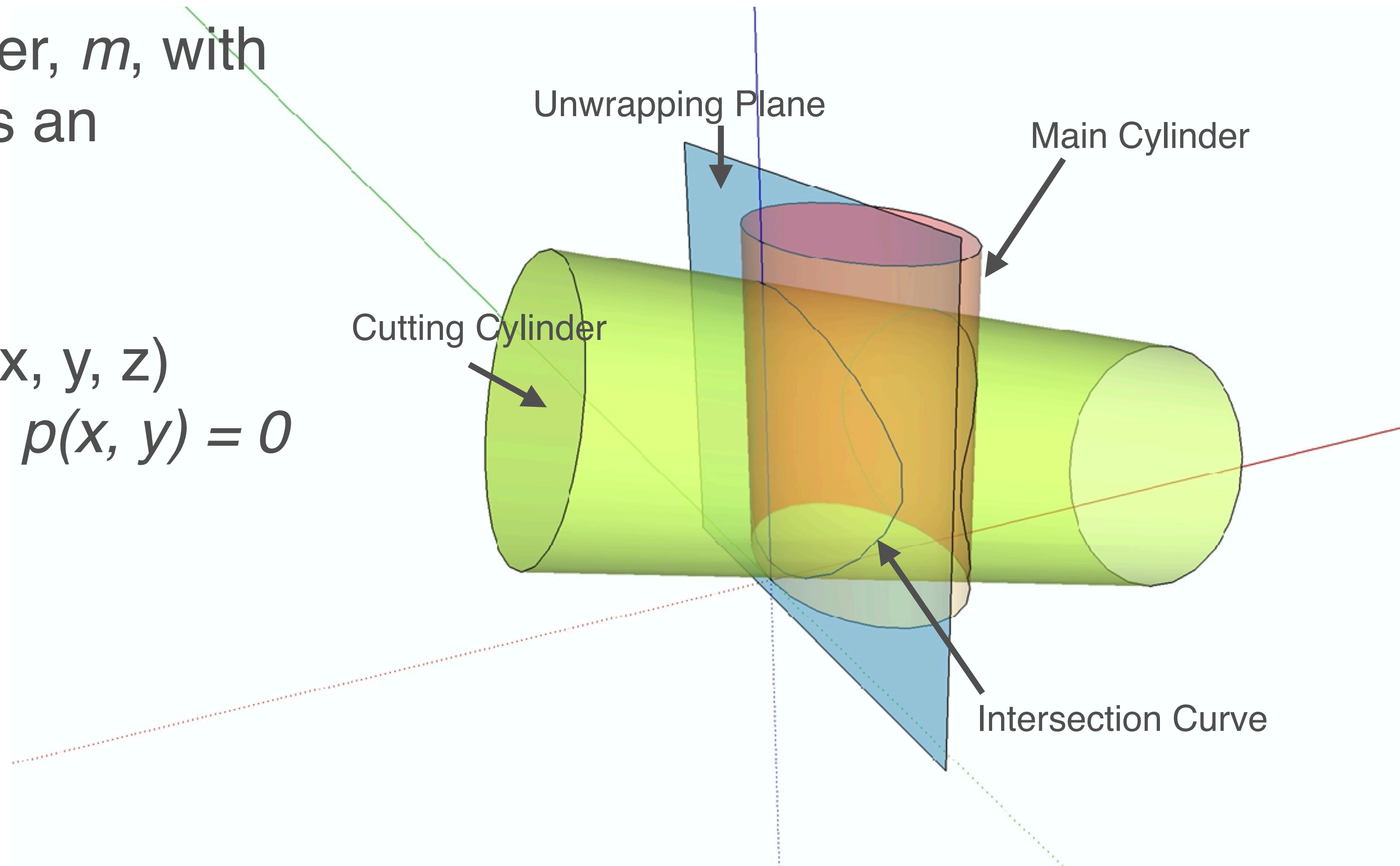
Curves on a Cylinder

- A curve C on the cylinder is projected onto a cutting plane in (x, z) . Call it C_u of form $u(x, z) = 0$.
- A cylinder is a *developable surface*, hence unwrapping it preserves distances. That is, the arc-lengths on profile gets projected onto a line segment with same length on the x -axis.



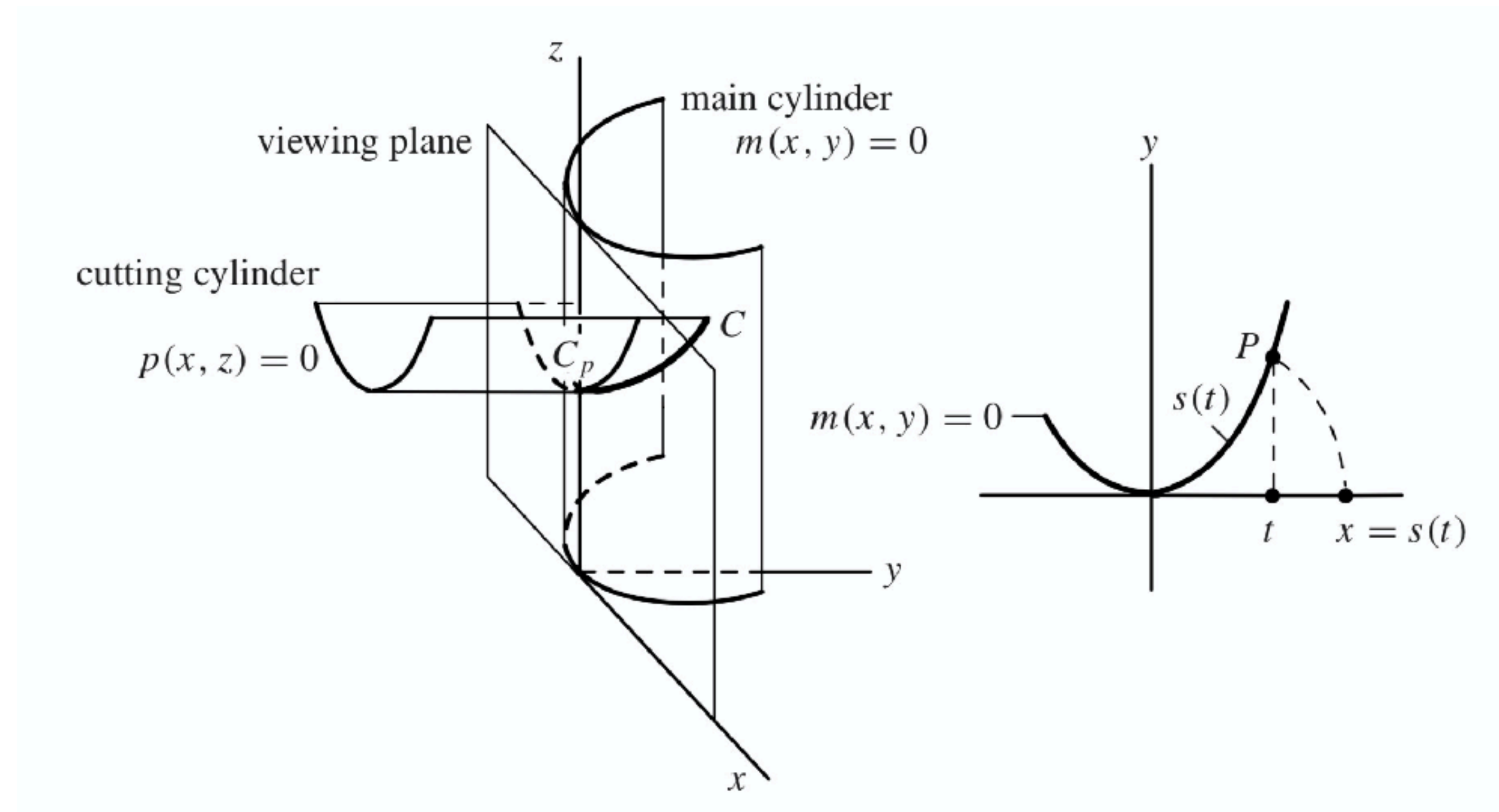
Parameter Description

- Intersection of Main Cylinder, m , with a Cutting Cylinder, p , yields an Intersection Curve, C .
- C is hence a set of points (x, y, z) where both $m(x, y) = 0$ and $p(x, y) = 0$



Parameter Description

- On the profile of Cutting Cylinder, C_p , each point P creates an arc-length s on the x -axis.
- For simplicity we define the Profile in terms of parameters t , and $s(t)$.
- These curves can be related via relations described next.



Relating Profile and Unwrapping Equations

- For a curve C , having horizontal profile equation $p(t, z) = 0$ for C_p and unwrapping equation $u(x, z) = 0$ for C_u , we have:

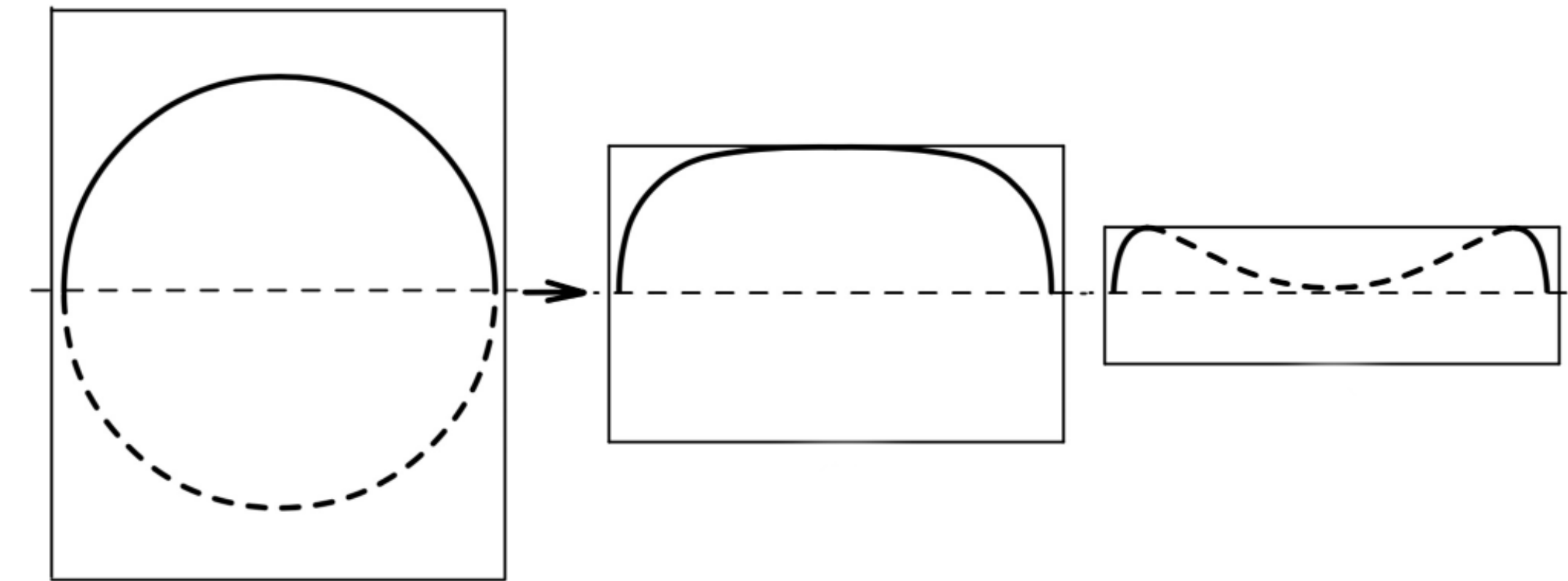
$$u(x, z) = p(s^{-1}(x), z)$$

$$p(t, z) = u(s(t), z)$$

- The relation above generalises the projection in orthogonal cutting planes.

Relating Profile and Unwrapping Equations

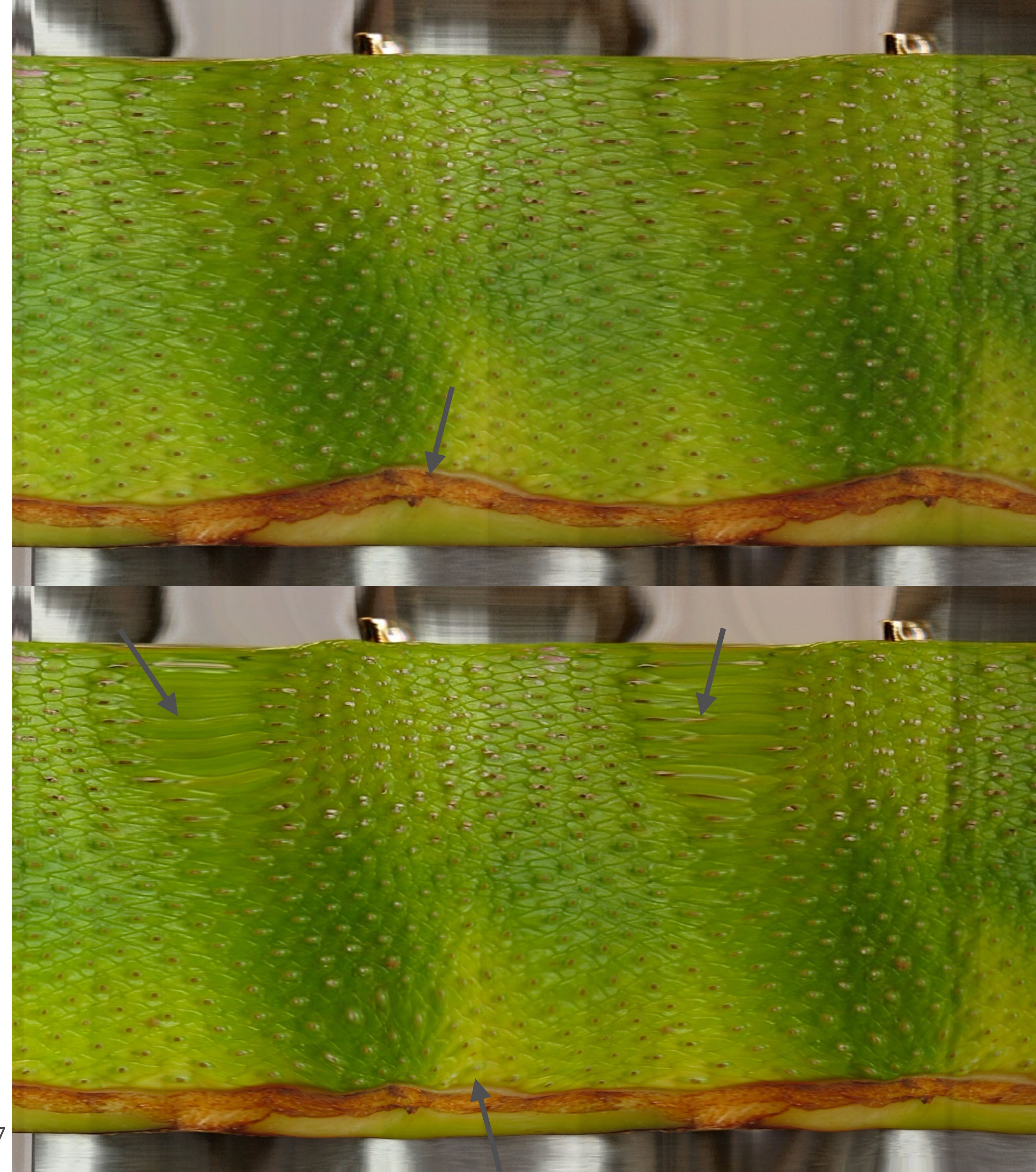
- This theorem gives us a relation between the profile equation and unwrapping equation.
- If the inverse of the arc-length function $s(t)$ exists, the relation is symmetric — hence, can be used to find the unwrapping equation from the profile.



#Cylinder Unrolling

Preliminary Progress

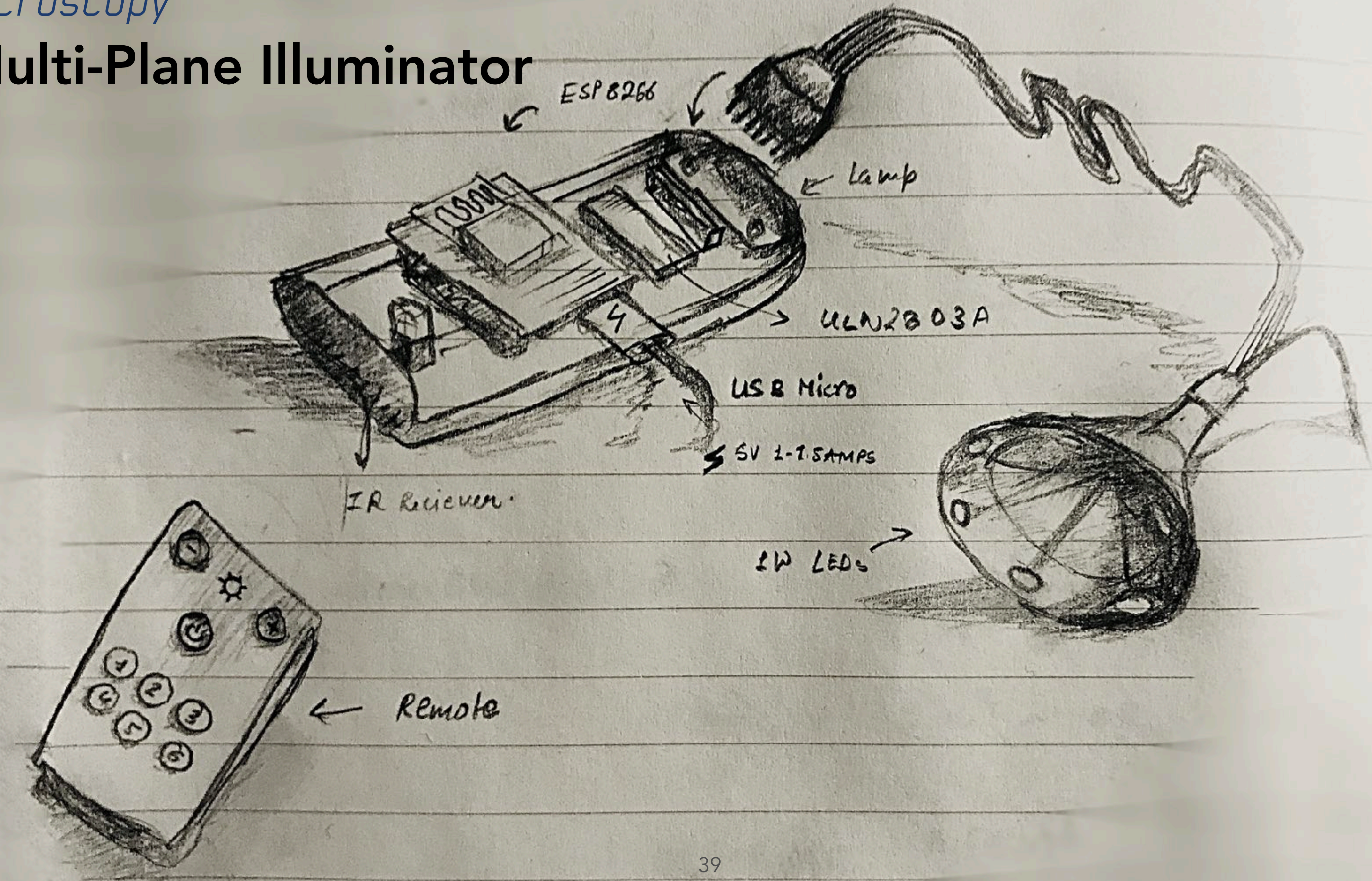
- The method works well at regular cylindrical surfaces.
- However, for spadix, not being an ideal cylinder as above, there's noticeable distortion on upper half of the image (where it "bends").
- Bottom half is correctly unwrapped.



#Microscopy

Multi-Plane Illuminator

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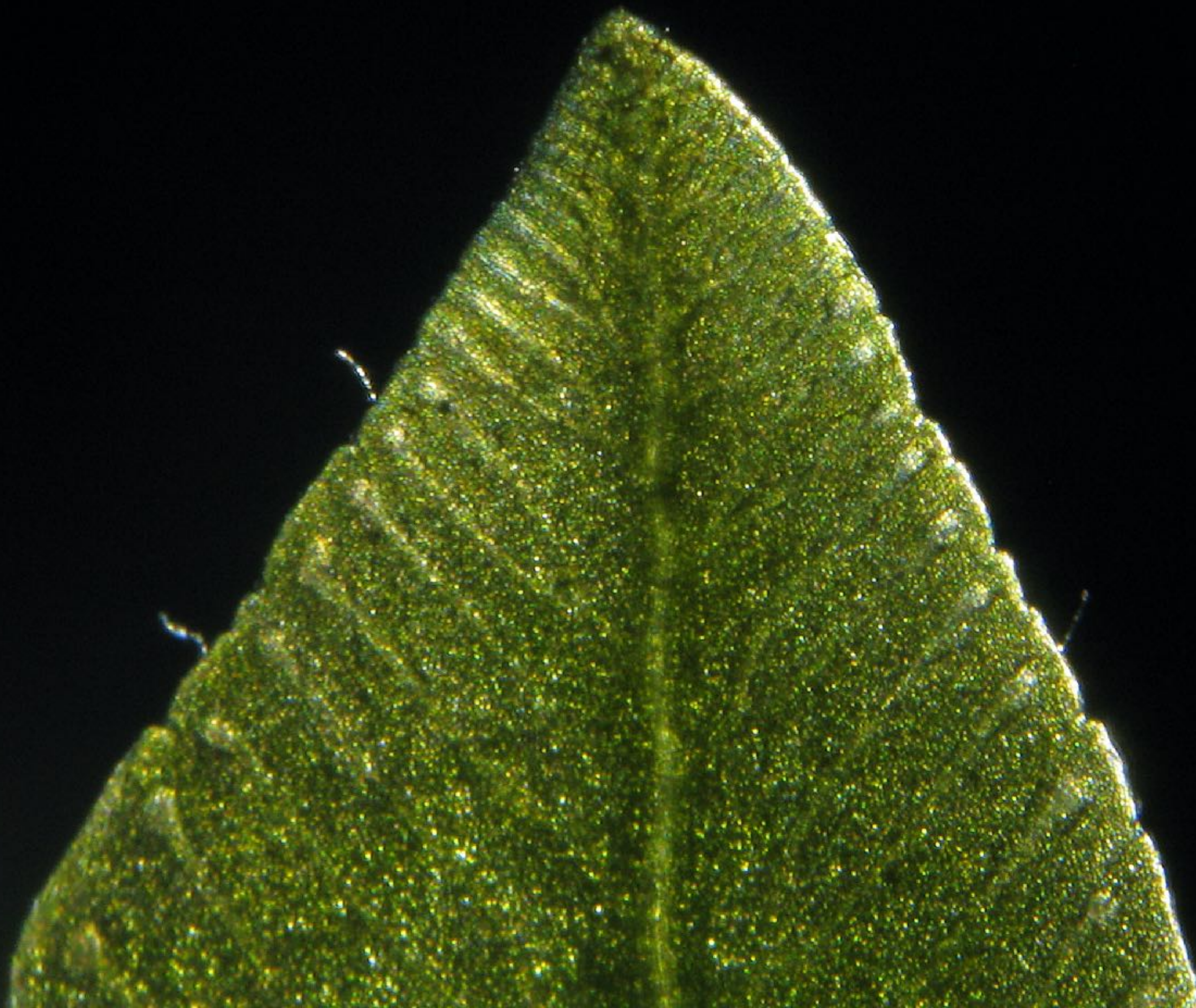


Microscopy

Multi-Plane Illuminator



20x



20x



20x



20x



#Summary of Activities

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- Anatomy of *Anthurium andreanum* spadix.

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- Affine transformation for unwarping the images.
- Cylinder unrolling.
- Multi-plane Illuminator.

Thanks!

